PRE A*-ALGEBRA AND HOMOMORPHISM

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ABSTRACT

This manuscript is a description of Pre A*- Homomorphism and explains the concept of kernel of Pre A*-homomorphism and established some theorems on these Pre A*-homomorphisms and its useful theorems. It distinguishes theorems related with these concepts of Pre A*-homomorphism.

KEYWORDS AND PHRASES: Pre A*-homomorphism, Pre A* - Algebra, Kernel of Pre A*-homomorphism, epimorphism, isomorphism, Pre A* - lattice

INTRODUCTION:

In a draft paper [3], The Equational theory of Disjoint Alternatives, around 1989, E.G. Manes introduced the concept of Ada (Algebra of disjoint alternatives) (A, \& V, (-)₁, (-)₂, 0, 1, 2)(Where \& V are binary operations on A, (-)₁, (-)₂ are unary operations and 0, 1, 2 are distinguished elements on A) which is however differs from the definition of the Ada of his later paper[4] Adas and the equational theory of if-then-else in 1993. While the Ada of the earlier draft seems to be based on extending the If-Then-Else concept more on the basis of Boolean algebras and the later concept is based on C-algebras (A, \&, V, (\neg)) (where \&, V are binary operations on A, (\neg) is a unary operation ) introduced by Fernando Guzman and Craig C. Squir[1]. In 1994, P. Koteswara Rao[2] first introduced the concept of A*-algebra (A, \&, V, *, (\neg), (\neg)₀, 0, 1, 2) (where \&, V, * are
binary operations on \( A \), \((-)\wedge\), \((-)\vee\) are unary operations and 0,1,2 are distinguished elements on \( A \) not only studied the equivalence with Ada, C-algebra, Ada’s connection with 3-Ring, Stone type representation but also introduced the concept of \( A^* \)-clone, the If-Then-Else structure over \( A^* \)-algebra and Ideal of \( A^* \)-algebra. In 2000, J.Venkateswara Rao\[5\] introduced the concept Pre \( A^* \)-algebra \((A, \lor, \wedge, (-)^{-})\) (where \( \wedge, \lor \) are binary operations on \( A \), \((-)^{-} \) is a unary operation on \( A \) analogous to C-algebra as a reduct of \( A^* \)-algebra, studied their subdirect representations, obtained the results that \( 2= \{0, 1\} \) and \( 3= \{0, 1, 2\} \) are the subdirectly irreducible Pre-A*-algebras and every Pre-A*-algebra can be imbedded in \( 3^X \) (where \( 3^X \) is the set of all mappings from a nonempty set \( X \) into \( 3= \{0, 1, 2\} \)). Praroopa.Y[7] introduced the specific concepts on Pre \( A^* \)-algebra and of the papers\[6],[8\], studied Pre \( A^* \)-algebra as a semilattice, lattice in Pre \( A^* \)-algebra

§PRELIMINARIES
Definition:

1.1 Pre \( A^* \)- Algebra [5] An algebra \((A, \lor, \wedge, (-)^{-})\) satisfying

\[
\begin{align*}
(a) \quad & (x^{-}) = x, \forall x \in A \\
(b) \quad & x \wedge x = x, \forall x \in A \\
(c) \quad & x \wedge y = y \wedge x, \forall x,y \in A \\
(d) \quad & (x \wedge y)^{-} = x^{-} \lor y^{-}, \forall x,y \in A \\
(e) \quad & x \wedge (y \wedge z) = (x \wedge y) \wedge z, \forall x,y,z \in A \\
(f) \quad & x \wedge (y \lor z) = (x \wedge y) \lor (x \wedge z), \forall x,y,z \in A \\
(g) \quad & x \wedge y = x \wedge (x^{-} \lor y), \forall x,y \in A
\end{align*}
\]

is called a Pre \( A^* \)- algebra.

1.2 Definition[7] Pre \( A^* \)- Homomorphism: Let \((A_1, \wedge, \lor, (-)^{-})\) and \((A_2, \wedge, \lor, (-)^{-})\) be two Pre \( A^* \) – algebras. A mapping \( f: A_1 \rightarrow A_2 \) is called an Pre \( A^* \) – homomorphism, if

\[
\begin{align*}
(i) \quad & f(a \wedge b) = f(a) \wedge f(b) \\
(ii) \quad & f(a \lor b) = f(a) \lor f(b)
\end{align*}
\]
(iii) \( f(a^-) = (f(a))^- \)

The homomorphism \( f : A_1 \to A_2 \) is onto, then \( f \) is called epimorphism.

The homorphism \( f : A_1 \to A_2 \) is one-one, then \( f \) is called monomorphism.

The homomorphism \( f : A_1 \to A_2 \) is one-one and onto then \( f \) is called an isomorphism,

and \( A_1, A_2 \) are isomorphic, denoted by \( A_1 \cong A_2 \).

1.3 Definition[7] Kernel of Pre A* - homomorphism:

By the definition of Pre A* - homomorphism, define Kernel of Pre A* - homomorphism

Let \( A_1, A_2 \) be two Pre A*-algebras and \( f : A_1 \to A_2 \) be a Pre A*-homomorphism then the set \( \{x \in A / f(x) = 0\} \) is called the Kernel of \( f \) and it is denoted by \( \text{Ker} f \).

1.4 Example:-

Let \( A \) be a Pre A*-algebra with 1, 0. Suppose that for every \( x \in A - \{0, 1\} \), \( x \vee x^- \neq 1 \). Define \( f : A \to \{0, 1, 2\} \) by \( f(1) = 1 \), \( f(0) = 0 \) and \( f(x) = 2 \) if \( x \neq 0, 1 \). Then \( f \) is a Pre A*-homomorphism.

1.5 Theorems on Pre A* -homomorphism[7]

Theorem: - Let \( f : A \to B \) be a Pre A*-homomorphism from a Pre A*-algebra \( A \) into a Pre A*-algebra \( B \) ad \( \text{Ker} f = \{x \in A / f(x) = 0\} \) is the Kernel then \( \text{Ker} f = \{0\} \) if and only if \( f \) is one-one.

1.6 Lemma[5]:

Let \( f : A_1 \to A_2 \) be Pre A*-homomorphism where \( A_1, A_2 \) are Pre A*-algebras with 1\text{1} and 1\text{2}-Then

(i) If \( A_1 \) has the element 2, then \( f(2) \) is the element of \( A_2 \).

(ii) If \( a \in B(A_1) \), then \( f(a) \in B(A_2) \)

where \( B(A_1) = \{x/x \vee x^- = 1\} \)
B(A_2) = \{ x/x \lor x \neg 1 \} 

1.7 Note:

If \( f: A \to B \) and \( g: B \to C \) are Pre A*-homomorphisms. So their composition or product \( \text{gof} : A \to C \), which is defined by \( \text{gof}(a) = g(f(a)) \) is also Pre A*-homomorphism.

1.8 Proposition[7]: If \( f: A \to B \) and \( g: B \to C \) are Pre A*-homomorphisms. Then

i) If \( f \) and \( g \) are mono, so is \( \text{gof} \)

ii) If \( f \) and \( g \) are epi, so is \( \text{gof} \)

iii) If \( \text{gof} \) is mono, so is \( f \)

iv) If \( \text{gof} \) is epi, so is \( g \)

1.9 Corollary[7]: The Pre A* homomorphisms \( f: A \to B \) is an isomorphism, if and only if, there exists a Pre A*-homomorphism \( g: B \to C \) such that \( \text{fog} \) is an automorphism of \( B \) and \( \text{gof} \) is an automorphism of \( A \).

1.10 Theorem[7]: Under any Pre A*-homomorphism \( f \) of a Pre A*-algebra \( A \) onto a Pre A*-algebra \( A_1 \) with 0, the set \( \text{kerf} \) (kernel of \( f \)) is an ideal in \( A \).

1.11 Proposition [7]: If \( f \) is a Pre A*-homomorphism of a Pre A*-algebra \( A \) into another Pre A*-algebra, then \( f(A) \cong A/f^1(0) \). Where \( f(A) \) is called the image,

\[ f^1(0) = \{ a \in A/ f(a) = 0 \} \text{ the Kernel of } f. \]

Conclusion: Established the concept of kernel of Pre A*-homomorphism and proved some theorems on these Pre A*-homomorphisms. Established its useful theorems and related with these concepts of Pre A*-homomorphisms.
REFERENCES