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MHD VISCO-ELASTIC BOUNDARY LAYER FLOW PAST A STRETCHING PLATE WITH HEAT TRANSFER

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1. ABSTRACT

MHD visco-elastic boundary layer flow problem past a stretching plate with heat transfer is investigated. The basic equations governing the flow and heat transfer are in the form of partial differential equations which have been reduced to a set of non linear ordinary differential equations by applying suitable similarity transfer. The effect of various physical parameter, magnetic parameter and prandtl number has been discussed. The results have been shown in different graphs.

Keywords: Visco-elastic liquid, Boundary layer flow, Stretching plate, Magnetic field, Similarity transformation, Heat transfer, Prandtl number.

2. INTRODUCTION

Interest in the flow and heat transfer of visco-elastic liquids has increased substantially over a past few decades due to number of applications in industrial manufacturing process like plasma physics, geophysics and in petroleum industry. Heat transfer in free and mixed convection in vertical channel occurs in many industrial processes. In the process of drawing artificial fibers, which is of great technical importance, it is governed by the rate at which the fiber is cooled and this in turn affects the final properties of the yarn. Many authors have done pioneering essential works in this regard.

Study of boundary layer behavior on continuous solid surface was studied by Sakiadis, B.C. [1] in 1961. Tsou, F.K. [2] et al. (1967), worked on the flow and heat transfer in the boundary layer on a continuously moving surface. Here they reported both analytical and experimental results for the flow and heat transfer aspects arising in stretching sheet. Rajgopal, K.R. [3-4] et al. (1984) studied the flow of visco-elastic fluids over a stretching sheet and got interesting results. They have further (1987) studied about a non-similar boundary layer flow on a stretching sheet by considering a non-Newtonian fluid with uniform free stream. The visco-elastic boundary layer flow past a stretching plate with heat transfer was studied by Ahmad, N. [5] et al. (1990). They have considered Walters liquid B and got interesting results relevant to heat transfer over stretching surface. A similar investigation [6] was made by them in 1991 where they studied about the Visco-elastic boundary layer flow past a stretching plate with suction and heat transfer. An important theoretical study was made in 1994 by Ariel, P.D. [7] on the flow of a Visco-elastic fluid past stretching porous plate. Chaim, T.C. [8] in 1996 has studied the heat transfer with variable conductivity in stagnation point flow towards a stretching sheet. Ahmad, N. [9] et al. (1999) have studied numerically about the visco-elastic boundary layer flow past a stretching plate and heat transfer with variable conductivity. They [10] have further (2000) worked on the visco-elastic boundary layer flow past a stretching plate with suction and heat transfer. They solved the problem numerically and got interesting results. Abel, M.S. [11] et al. (2001) have studied the heat transfer in visco-elastic fluid flow over a stretching surface where they have investigated the effect of magnetic field on flows of non-Newtonian fluids. A similar investigation has been made by Subhas, A. [12] et al. (2005) where they studied the buoyancy force and thermal radiation effects in MHD Visco-elastic boundary layer flow over a continuously moving stretching surface. Abel, M.S. [13] et al. (2007) have worked on the heat transfer in a visco-elastic boundary layer flow over a stretching sheet with viscous dissipation and non uniform heat source. Ahmad, N. [14] et al.(2010) have also studied the boundary layer flow and heat transfer past a stretching plate with variable thermal conductivity. In 2011 Ahmad, N. [15] has studied the visco-elastic boundary layer flow past a stretching plate and heat transfer with variable thermal conductivity. He has studied the problem in two cases namely 1) Prescribed surface temperature (PST) and 2) Prescribed stretching plate heat flux (PHF). In this work he has shown when visco-elasticity increases, the fluid absorbs more heat as result temperature increases. He has also shown that when Prandtl number (Pr) increases, temperature decreases. A similar investigation was made by Mishra, M. [16] et al. (2012), where they studied about an unsteady boundary layer flow of viscous incompressible fluid over a stretching plate. They solved the heat flow problem with variable conductivity. The velocity field has been obtained by similarity transformation method and heat flow problem was studied by considering both PST and PHF cases. A study of MHD boundary layer flow with variable viscosity over a heated stretching sheet was made by Hassen, H.S [17] et al. (2015) by using Lie-Group method. In this paper, we have studied the MHD visco-elastic boundary layer flow past a stretching plate with heat transfer by using similarity transformation method. The heat transfer problem is solved by considering prescribed power law surface temperature. In the way the present paper is an extension of the work done by Ahmad, N. [15].

3. MATHEMATICAL FORMULATION AND SOLUTION

The problem considered here is the study of boundary layer flow due to a moving flat plate in a visco-elastic electrically conducting fluid in presence of a uniform magnetic field B which is normal to the surface. The flow is two dimensional where X-axis is along the plane of moving plate and y-axis is normal to it, respectively. We assume that the surface is moving continuously with the velocity $u_w = U_w(x)$ in the positive x-direction. We denote the external velocity by u_e and we assume that $u_e(x) = u_\infty x^{-1}$ with $u_\infty > 0$. Under these assumptions, the boundary layer along moving plate is governed by the equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\} + \frac{\sigma B^2(x)}{\rho}(u_e - u) \tag{2}$$

Where u, is the horizontal velocity component; v, the vertical velocity component; k_0, the coefficient of visco-elasticity; ρ, the fluid density and σ, the electric conductivity. The relevant boundary conditions are:

$$y = 0, u = u_w = mx, v = 0, m > 0$$
$$y \to \infty, u = 0$$

Introducing the dimensionless variables

$$\bar{y} = \frac{y}{h}, \bar{u} = \frac{uh}{\nu}, \bar{x} = \frac{x}{h}, \bar{v} = \frac{vh}{\nu}$$

The equations 1 & 2 reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - k_1\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\} + \frac{\sigma B^2(x)}{h^2\rho}(u_e - u) \tag{4}$$

Where $k_1 = \frac{k_0}{h^2}$ and bars has been dropped for convenience.

Boundary conditions are now

$$y = 0, u = mx, v = 0, m > 0$$
$$y \to \infty, u = 0 \tag{5}$$

Setting the similarity solution of the form $u = mxf'(y)$, we have

$$v = -mf(y) \tag{6}$$

Putting u and v in equation-2, we have

$$f'^2(y)-f''(y)f(y)=\frac{1}{m}f'''(y)-k_1\{2f'(y)f'''(y)-f(y)f^{iv}(y)-f''^2(y)\}-\frac{M_0}{m}f'(y) \tag{7}$$

Where $M_0=\dfrac{\sigma Bh^2}{\rho v^2}$.

which is a non linear differential equation of order four.
The boundary conditions (5) reduce to

$$y=0,\ f'=1,\ f=0$$
$$y\to\infty,\ f'=0$$

Boundary conditions suggest that the velocity function may be of the form

$f'(y)=e^{-ry}$, where r is a complex number with positive real part.

Thus $\quad v=-\dfrac{m}{r}\left(1-e^{-ry}\right)$

Now from equation (7), we get

$$r=\frac{1}{\sqrt{\frac{1}{m}-k_1}}$$

Therefore, the velocity components become as follows:

$$u=mxe^{-ry}\ \&\ v=-\frac{m}{r}\left(1-e^{-ry}\right) \tag{8}$$

4. HEAT TRANSFER PROBLEM

In absence of viscous dissipation and heat generation, the energy equation for two dimensional heat flows is given by

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\sigma B^2u^2$$

$$\Rightarrow\quad u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\sigma B^2u^2}{\rho C_p} \tag{9}$$

Subject to the boundary conditions

$$y=0, T=T_P$$
$$y\to\infty, T=T_\infty \tag{10}$$

Where T_P is the plate temperature, T_∞ is temperature of the surrounding fluid, C_p I specific heat at constant pressure and k is thermal conductivity.

Let the surface temperature be of the form

$$y=0, T=T_p=T_\infty+A\left(\frac{x}{l}\right)^2$$

While the temperature outside the dynamic region be $y\to\infty, T=T_\infty$. Now we define the dimensionless temperature by

$\psi(\eta)=\dfrac{T-T_\infty}{T_p-T_\infty}$, where

$\eta = ry$.

For conducting fluids, it has been found that the thermal conductivity varies with temperature in an approximately linear manner in the range $0^0 F to 400^0 F$. Therefore we assume k as $k = k_\infty (1+\varepsilon\psi)$ where $\varepsilon = \dfrac{k_p - k_\infty}{k_\infty}$.

Now substituting u and v and changing the independent variable y to $\eta = ry$, we have

$$\psi'' + \frac{m\Pr}{r^2}\left(1-e^{-\eta}\right)\psi' + \varepsilon\left(\psi\psi'' + \psi'^2\right) = 0 \tag{11}$$

The corresponding boundary conditions are

$$\eta = 0, \psi = 1 \tag{12}$$

$$\eta \to \infty, \psi = 0 \tag{13}$$

From equation (11), we see that the heat transfer takes parts in two parts, i.e. one part of heat transfer is due to temperature difference and the other part is due to variable thermal conductivity. We denote the first part by ψ_m and the second part by ψ_v. Thus equating the terms independent of ε and the terms involving ε, we have

$$\psi_m'' + \frac{m\Pr}{r^2}\left(1-e^{-\eta}\right)\psi' = 0 \tag{14}$$

$$\psi_m(0) = 1, \psi_m \to 0 \, as \, \eta \to \infty \tag{15}$$

$$\psi_v \psi_v'' + \psi_v'^2 = 0 \tag{16}$$

$$\psi_v(0) = 1, \psi_v \to 0 \, as \, \eta \to \infty \tag{17}$$

The solution of equation (14) is

$$\psi_m = \frac{\int_\eta^\infty \exp\left\{-\frac{m\Pr}{r^2}\left(\eta + e^{-\eta}\right)\right\} d\eta}{\int_0^\infty \exp\left\{-\frac{m\Pr}{r^2}\left(\eta + e^{-\eta}\right)\right\} d\eta} \tag{18}$$

Equation (17) is a non-linear differential equation of order two. Let the solution of this equation be of the form

$\psi_v(\eta) = (1-\eta)^\alpha$

Putting this solution in equation (18), we have

$$2\alpha^2 - \alpha = 0 \tag{19}$$

The roots of this equation are 0 and ½. Therefore,

$$\psi_v(\eta) = A + B\sqrt{1-\eta} \tag{20}$$

The general solution (20) of equation (16) is real only when $0 \le \eta \le 1$. Therefore the heat transfer due to variable thermal conductivity takes place within the dynamic region $0 \le \eta \le 1$. Hence the boundary conditions (17) may be presented as

$$\begin{aligned} &\eta = 0, \psi_v = 1 \\ &\eta \to \infty, \psi_v \to 0 \end{aligned} \tag{21}$$

The solutions (20) finally reduces to

$$\psi_v = (1-\eta)^{\frac{1}{2}} \forall \eta \in [0,1] \tag{22}$$

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5. DISCUSSION OF THE RESULTS

In this problem, we have studied MHD visco-elastic boundary layer flow past a stretching plate with heat transfer by using similarity transformation method. The effect of visco-elastic parameter and magnetic parameter on velocity profile and effect of prandtl number on temperature distribution has been studied.

Fig.1 depicts the effects of magnetic parameter M_0 on the velocity profile for different values of visco-elastic parameter k_1. From the figure, it is clear that velocity profile decreases monotonically to zero as y increases from the boundary.

In Fig-2, we see the effect of prandtl number on temperature distribution in the absence or presence of the magnetic parameter M_0. The higher prandtl number fluid causes a fall in temperature due to low thermal diffusivity

In Fig-3, we see the effect of visco-elastic parameter k_1 on the mean temperature field $\psi_m(\eta)$ in PST case. It is seen that the temperature $\psi_m(\eta)$ at the surface of stretching plate is invariant with respect to the other physical parameters. This coincides with the result of [15].

Fig.4 depicts the mean temperature field $\psi_m(\eta)$ with regards to the prandtl number $\Pr$ in PST case. From this figure, it is observed that the mean temperature decreases as the prandtl number $\Pr$ increases.

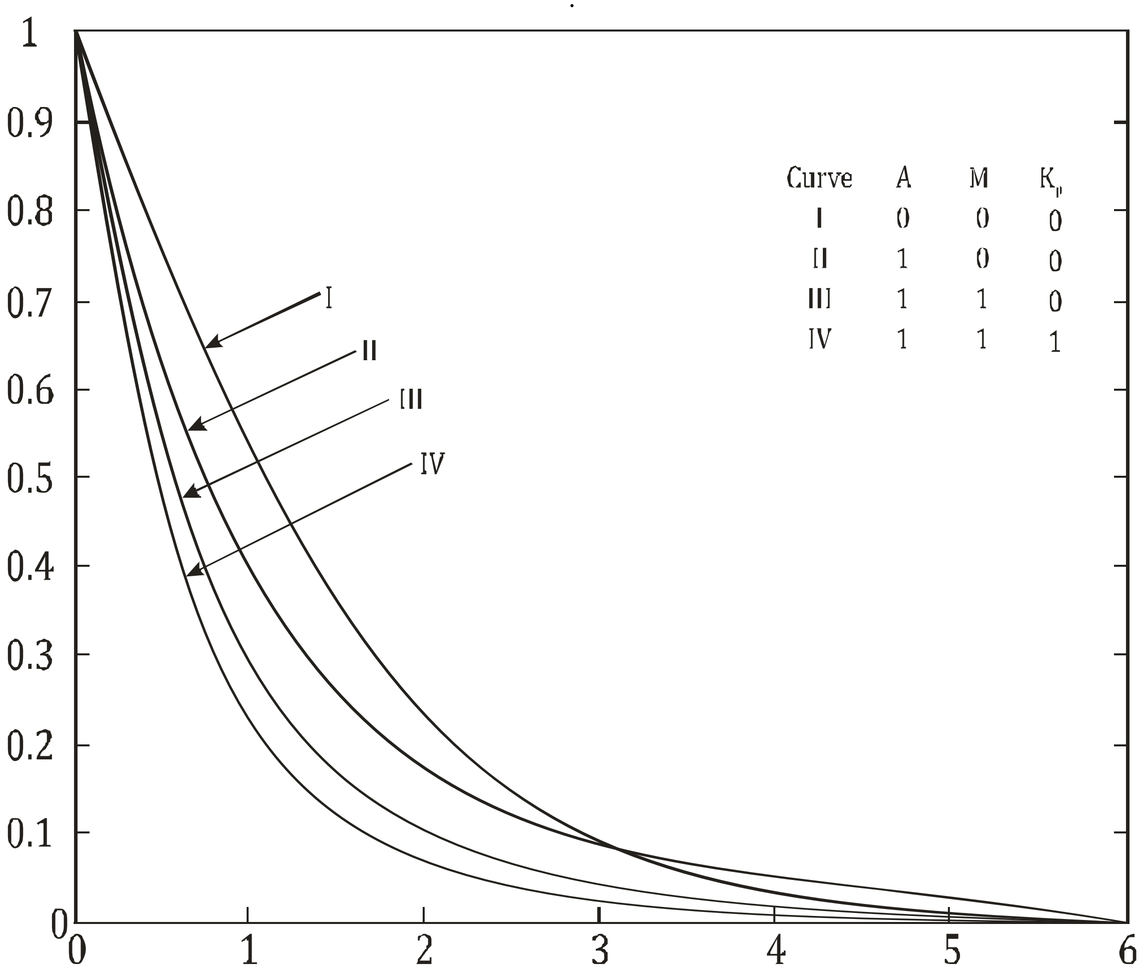

Figure 1: Effects of M_0 and k_1 on Velocity Profile.

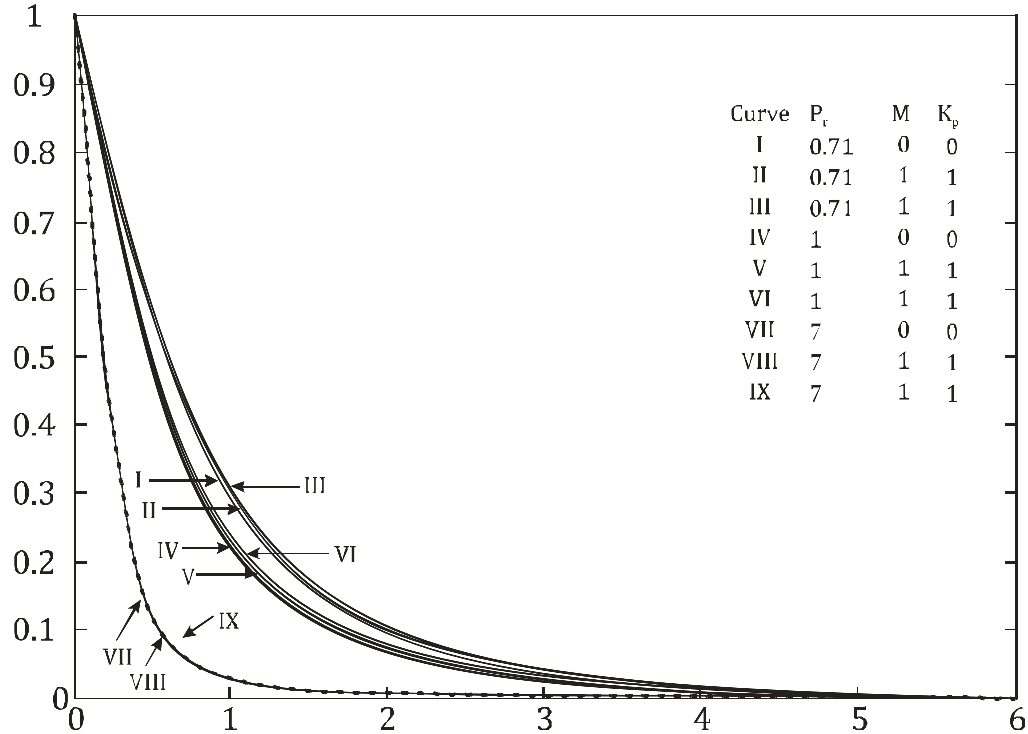

Figure 2: Effects of Prandtl Number on Temperature Distribution.

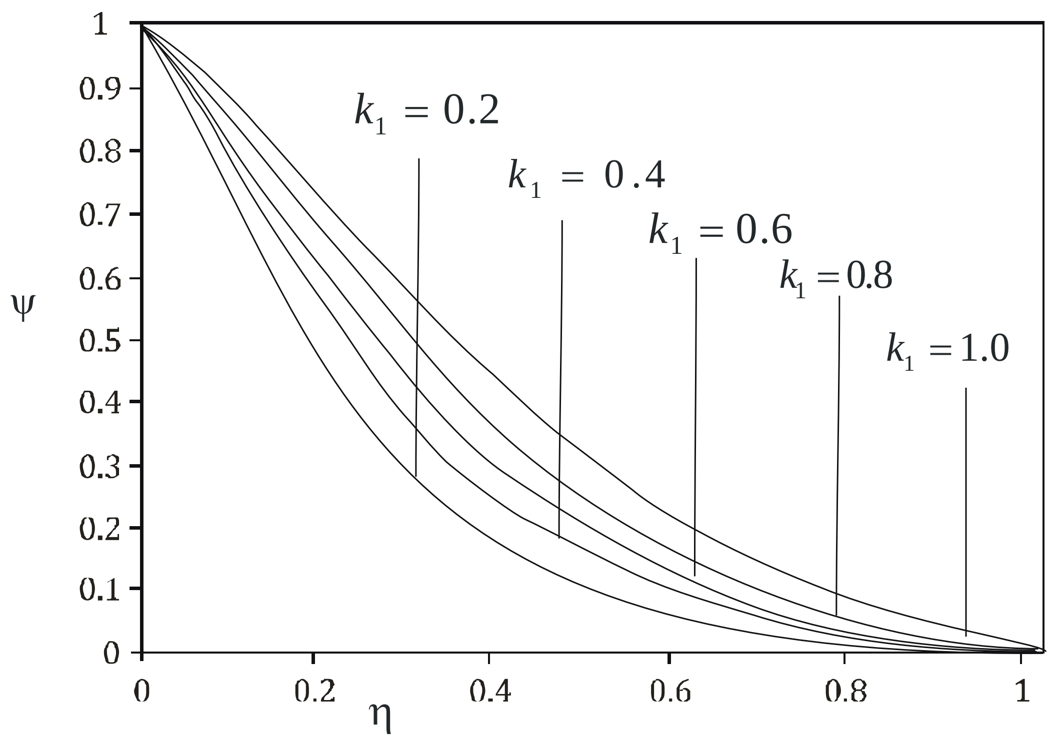

Figure 3: Mean Temperature $\psi_m(\eta)$ for different values of k_1.

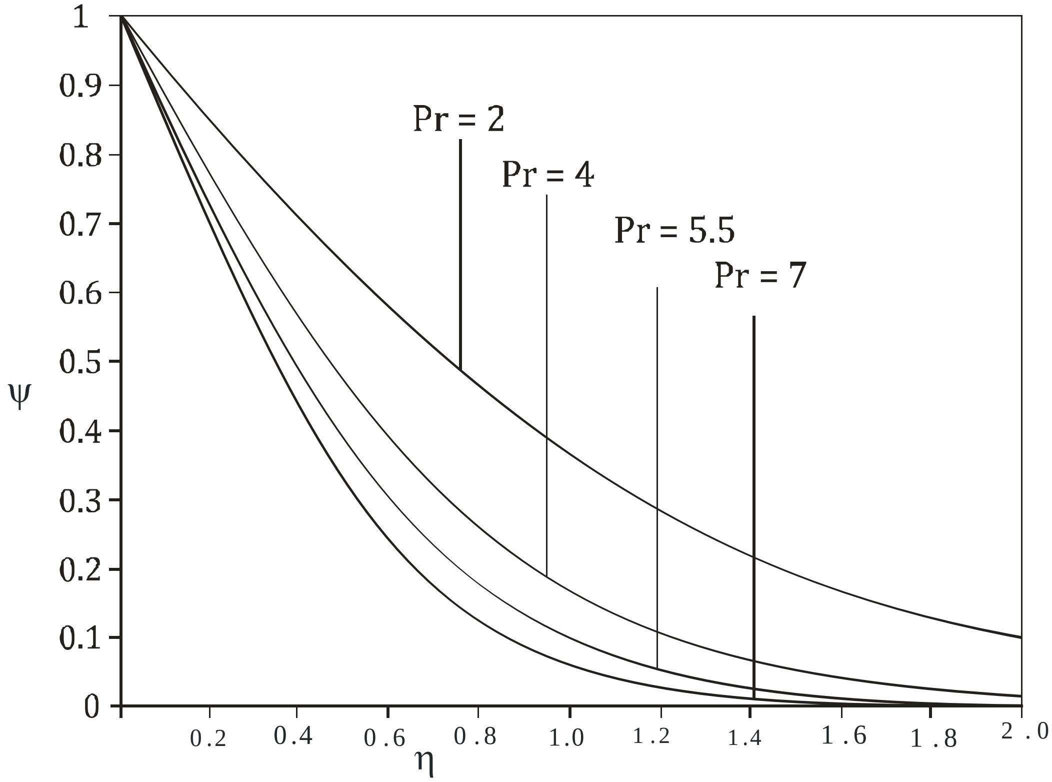

Figure 3: Effects of Prandtl Number Pr on Mean Temperature $\psi_m(\eta)$ with fixed $k_1 = 0.04$.

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