Low Complexity Adaptive DOA Algorithm

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ABSTRACT
Beam forming is a technique that equips the system with a strengthened response in a specific direction while evading noise, interference and jamming from other directions. Ahead of beamforming the directions of users and interference must be procured using a direction of arrival estimation algorithm (DOA). The DOA algorithm for estimation measures the angle of arrival of all the impinging signals from the receiving signals through each antenna element of the array. This paper includes simulation of MUSIC and ESPRIT DOA algorithms, in addition to simulation of Adaptive Beamforming using LMS algorithm using Matlab platform as simulator.

Keywords
Digital beamforming, MUSIC, ESPRIT, Adaptive beamforming, spatial filtering.

INTRODUCTION
Beamforming is a substitute label for spatial filtering where, with pertinent analog or digital signal processing, an array of antennas can be steered in a way to arrest the reception of radio signals coming from explicit directions. While a filter in the time domain incorporates energy over time, the beamformer incorporates energy over its aperture, acquiring a certain antenna gain in a given direction while having debilitation in others. Beamforming has been exploited for many years in diverse applications in the aspects of radio communication such as surveillance, radar and, with different array sensors, in sonar and audio fields. Beamforming is typically adept by phasing the feed to each element of an array in a way that signals received or transmitted from all elements will be in phase in a distinct direction [7]. Beamforming is used along with an array of antennas/sensors to transmit/receive signals to/from a specified spatial direction in the presence of interference and noise [6].

BEAMFORMING
The effects of co-channel interference, multipath fading and background noise can be abridged by employing multiple antennas in a receiver. An array structures an improved estimate of the desired signal by weighting and summing the signals received at multiple spatially distant antennas. High gain can be placed in the direction of a desired signal and low gain can be placed in the direction of interfering signals by aptly choosing the weights. This course is referred to as beam forming or spatial filtering.

The weighting applied to the signals received at each antenna may be fixed or may be constantly adjusted to track variations in the signal environment. A radiation pattern of the antenna array is devised by
Beam forming via adding the phases of signals in the desired direction and nulling the pattern in the unwanted direction. The phases (the inter element phase) and often the amplitudes are altered to optimize the received signal. Beamforming can be categorised as fixed and adaptive beamforming [4].

**Conventional/fixed Beamforming**

In a conventional beamformer the weights are pre-calculated and are exclusive of the arriving data. The delays experienced in each sensor due to path difference are basically the weights, so that the outputs of spatially distributed sensors are systematically summed to improve signal reception in the proximity of noise [6].

![Figure 1: Conventional beamformer](Image)

**Adaptive Beamforming**

In Adaptive beamformer, optimization of the array pattern according to the changing electromagnetic environment is brought about as a result of an adaptive algorithm. An adaptive array system consists of antenna array elements terminated in an adaptive processor which is designed to precisely maximize certain criteria. As the emitters move or change, the adaptive array updates and improves iteratively in order to trail the changing environment [5].

The signal processing unit computes the weight vector according to a specific control algorithm when supplied by the data samples collected by antenna array [1].

The array output equation is \( y(t) = w^H x(t) \).  

![Figure 2: Adaptive beamformer](Image)
DIRECTION OF ARRIVAL ESTIMATION ALGORITHMS

Direction finding or DOA estimation can be outlined as the estimation of the wave number or angle of arrival of a plane wave. Antenna arrays are widely used to figure out direction finding [6].

To estimate the signal arriving from a specific direction in the presence of noise and interfering signals, a receive beamformer is customarily used. The DOA estimation algorithms under consideration are MUSIC and ESPRIT.

Multiple signal classification (MUSIC)

MUSIC is one of the primal method proposed by Schmidt and a very prominent method for super-resolution direction finding [3].

MUSIC algorithm can be summarized as follows [3]:

1. Collect input samples \( x_n(t), n = 1, 2, ..., N \) and calculate the input covariance matrix
   \[
   R_x = \frac{1}{N} \sum_{n=1}^{N} x_n(t) x_n^H(t) = A R_s A^H + \sigma_N^2 I_N
   \]
   where \( R_s \) is the signal covariance matrix, \( \sigma_N^2 \) is the noise common variance and \( I_N \) is the identity matrix of rank \( N \), \( A \) is collection of the steering vectors of the array.

2. Performing eigen decomposition on \( R_x \)
   \[
   R_x E = EE^H
   \]

3. Estimate the multiplicity \( k \) of the smallest eigenvalues \( \lambda_m \) and then the number of signals \( d \) from as \( d = N - k \).

4. Compute the MUSIC spectrum
   \[
   P(\theta) = P_M(\theta) = \frac{1}{a(\theta) a^H(\theta) E_n E_n^H a(\theta)}
   \]
   where \( E_n = [q_{d+1}, ..., q_N] \) with \( q_i, i = d + 1, d + 2, ..., N \).

Estimation of signal parameters via rotational invariance technique (ESPRIT)

Although the performance advantages of MUSIC are ample, they are attained at an extensive cost in computation and storage. Momentous reduction in the aforementioned computation and storage costs is brought about by ESPRIT. This is done by instituting constraints on the structure of the sensor array to possess displacement invariance, i.e., sensors take place in matched pairs with identical displacement vectors.

ESPRIT algorithm infers that the \( N \)-element array is comprised of two identical translated \( N' \)-element subarrays, where \( N' < N \leq 2N' \), as depicted in the figure below. The elements in each pair of identical sensors, or doublet, are considered to be separated by a fixed displacement vector \( D \).

Figure 3: ESPRIT sensor array geometry: Two overlapping subarrays

For certain special array structures, the subarray may overlap, i.e., an array element may be a member of \( (N < 2N') \) as shown in figure above. For subarrays that do not share elements, \( N = 2N' \) as shown in figure below [1].
ESPRIT algorithm can be summarised as [5]:

- Compute the array correlation matrices $R_{11}$ and $R_{22}$ from the data samples.
- After knowing the array correlation matrices from both subarrays, the total number of sources can be calculated by the largest eigenvalues either in $R_{11}$ or $R_{22}$.
- Determine the signal subspace $E_1$ and $E_2$ based upon the signal eigenvectors $R_{11}$ and $R_{22}$.
- Model a 2d x 2d matrix using the signal subspaces such that

$$C = \begin{bmatrix} E_1 & E_2 \end{bmatrix} = E_C \Lambda E_C^H \quad (5)$$

where the matrix $E_C$ is from the eigenvalue decomposition of $C$ such that

$$\Lambda = \begin{bmatrix} \lambda_1, \lambda_2, ..., \lambda_{2d} \end{bmatrix}, \quad \lambda_1 \geq \cdots \geq \lambda_{2d}.$$  

- Section $E_C$ into four d x d matrices such that

$$E_C = \begin{bmatrix} E_1 & E_2 \end{bmatrix} \quad (6)$$

- Calculate the rotational operator $\Psi = -E_1 E_2^{-1}$.  
- Estimate the eigenvalues of $\Psi$, $\lambda, \lambda_1, \lambda_2, ..., \lambda_d$.
- Compute the angles of arrival, given that

$$\theta_i = \sin^{-1}\left(\frac{\lambda_i}{2\pi D} \frac{a}{\lambda_i} \right), \quad i = 0, 1, ..., K. \quad (8)$$

### ADAPTIVE BEAMFORMING ALGORITHMS

Exploiting the information furnished by the DOA estimation algorithm, the adaptive algorithm computes the apt complex weights to cast the maximum radiation of the antenna pattern towards the desired user and sets the nulls towards the directions of the interferers.

Adaptive Beamforming algorithms can be classified into two categories: non-blind adaptive algorithms and blind adaptive algorithms. Non-blind adaptive algorithms count on statistical knowledge about the transmitted signal in order to converge to a solution. This is typically carried out by employing a pilot training sequence sent over the channel to the receiver to help it identifying the desired user. Further, blind adaptive algorithms do not require prior training, and hence they are attributed as “blind” algorithms. These algorithms attempt to elicit salient characteristic of the transmitted signal in order to sever it from other users in the surrounding environment [4]. The adaptive algorithm under consideration is least mean squares algorithm.
Least mean squares (LMS) algorithm
The least mean squares algorithm is a gradient based technique. Gradient based algorithms assume a settled quadratic performance surface. The error can be indicated as

$$\epsilon(k) = d(k) - w^H(k) \chi(k). \quad (9)$$

The squared error is given by

$$|\epsilon(k)|^2 = |d(k) - w^H(k) \chi(k)|^2. \quad (10)$$

The cost function is given by

$$J(w) = D - 2w^H \tau + w^H R_\chi w \quad (11)$$

where $D = E[|d|^2]$.

Gradient method is engaged to locate the minimum of equation. Thus

$$\nabla_w[J(w)] = 2 R_\chi w - 2 \tau. \quad (12)$$

The minimum occurs when the gradient is zero. Thus the solution for weights is the optimum Weiner solution as given by

$$w_o = R_\chi^{-1} \tau. \quad (13)$$

The method of steepest descent can be estimated in terms of weights using the LMS method. The steepest descent iterative approximation is given by

$$w(k+1) = w(k) - \mu \nabla_w J(w(k)) \quad (14)$$

where $\mu$ is the step-size parameter and $\nabla_w$ is the gradient of performance surface. The LMS solution is given by

$$w(k+1) = w(k) - \mu [R_\chi w - \tau]$$

$$w(k+1) = w(k) + \mu \epsilon^*(k) \chi(k) \quad (15)$$

where $\epsilon(k)$ is the error signal.

The convergence of LMS algorithm is directly proportional to the step-size parameter $\mu$. If the step size is too small, convergence is slow and we have over-damped case. If the step size is too large, LMS algorithm will overshoot the optimum weights of interest. This is called under-damped case.

If aimed to converge too fast, the weights will oscillate about the optimum weights but will not accurately record the solution desired. It is therefore obligatory to choose a step size in a range that safeguards convergence. Stability is assured provided the following condition is met

$$0 < \mu < \frac{1}{2 \lambda_m} \quad (16)$$

where $\lambda_m$ is the largest eigen value of $R_\chi$. It requires about 2N complex multiplications per iteration, where $N$ is the number of weights (elements) used in the adaptive array [5].

**SIMULATION AND RESULTS**

The simulation of MUSIC algorithm is carried out in Matlab. The three Directions of arrival detected are 20, 30, 60 degrees.
The simulation of ESPRIT algorithm is carried out in Matlab. The three Directions of arrival detected are 20, 30, 60 degrees.

The following results indicate the array response of 8-element ULA using LMS algorithm and its error plot. Here the main beam is pointed in the direction of desired signal that maximizes its reception while nulling the interference signal by creating nulls in its direction. Number of side lobes depends upon the number of elements in the array. By increasing the number of elements in uniform linear array, we can increase directivity of the array by achieve more narrow beam but number of side lobes also increases that causes the loss of power.
Figures 7 & 8 show the beam plotting of the desired signal at 20° and nulling the interfering signal at 60 degrees and 30 degrees. This indicates the spatial separation of desired signal from interfering signal.

CONCLUSION

Simulations are carried out for MUSIC and ESPRIT to estimate the DOA and it is found that ESPRIT is advantageous over MUSIC in terms of cost and storage. Simulations are also carried out for Adaptive beamforming using LMS algorithm. LMS algorithm is significant considering its simplicity and ease of computation.
REFERENCES


